(Rubel) Is the ring (or algebra) of entire functions of order $p$ isomorphic to that of order $p'$ if $p \neq p'$?

Answer is no by E.G. Straus.
(Rotman). If $A$ is an abelian group, set $A^* = \text{Hom}(A, \mathbb{Z})$, $\mathbb{Z}$ = integers

Is $A^* \cong A^{**}$?
(Granger): If a group contains a free subsemigroup with two generators, does it necessarily contain also a free subgroup with two generators?

This question is connected with the following question on amenable semigroups: If $G$ is an amenable group, then is every subsemigroup also amenable?

A.H. Feit proved that a group contains a free subsemigroup with two generators if and only if every subsemigroup of any two right ideals have nonvoid intersection.

And also: If a left amenable group does not contain the free subsemigroup with two generators, then every subsemigroup is left amenable.

Let $G = \{a, b : a^6ab^2 = e\}$. The semigroup on $a, b$ is free.

[Appel-Popov]
(Rubel) Let \( B \) be a vector subspace (\( B \) is not assumed to be closed) of \( L^2(\mathbb{R}, \mathbb{R}) \) such that if \( f \in B \) and \( |g(x)| \leq |f(x)| \) a.e. then \( g \in B \), and such that if \( f \in B \) then \( f^n \in B \), where \( f^n \) is the Fourier transform of \( f \). Is it true that either \( B = L^2 \) or \( B = \{0\} \)?

Solution: No. Let \( \mathcal{P}_{\mathbb{R}/0} \) be the class of finite linear combinations of positive definite functions.
Can every torsion-free abelian group be provided with an archimedean lattice-order?

This will not be the case if there exists a torsion-free abelian group of cardinality greater than \( c \) with the property that every pure subgroup is indecomposable.

There are none.

No complete solution to be published by Griffith in Proc. A.M.S.
Let $F$ be a family of functions $f$ holomorphic in the unit disc, such that for each $z$ in the disc,

\[ \{ f(z) : f \in F \} \text{ is countable.} \]

Is the family $F$ countable?

Dixmier has a short proof that the continuum hypothesis is true if there exists an uncountable family.

He shows that if the continuum hypothesis is false, then each family is countable.

If $\kappa = \aleph_1$, such a family can have power $\kappa$ (Cichoń's)

The following problem remains: Assume that for every $\alpha \in F$, $f \in F_3$ has power $< \kappa$. Is the family $F$ of power $< \kappa$? If $\kappa = \aleph_1$, this is false.

What happens if $\kappa > \aleph_1$?
It might be to someone's benefit to consider the situation for a UF domain which properly contains the ring of rational integers.
(Erdös and Heilbronn) Let $a_1, a_2, \ldots, a_k$ be $k$ distinct residues $(\mod p)$. Is it true that the number of distinct residues of the form $a_i + a_j, i \neq j$, is at least $\min(2k-3, p)$?
(Endōs) Are there for every $\epsilon > 0$ more than $m(1-\epsilon)$ integers $1 < a_1 < a_2 < \ldots < a_k \leq n$, $k > m(1-\epsilon)$ so that if $a_{i_1}, a_{i_2}, \ldots, a_{i_r} = a_{j_1}, a_{j_2}, \ldots, a_{j_l}$, $i_1 < i_2 < \ldots < i_r$; $j_1 < j_2 < \ldots < j_l$ then $r = l$?

For example, the numbers $2(2m+1)$ have this property, but their number is only $m/4$.

Selfridge showed that if $n$ is sufficiently large one can give $\frac{m}{\epsilon} (1-\epsilon)$ such integers.
(Erdős) Let $a_i \pmod{m_i}$, $1 \leq i \leq k$ be such that there is an integer $U$ for which $U \neq a_i \pmod{m_i}$, $1 \leq i \leq k$. Then this integer can be chosen to satisfy $0 < U \leq 2^k$. This sharpens a conjecture of Stein. He assumes $n_1 < \ldots < n_k$ and that no $x$ satisfies two of the congruences (it then follows that there is a $x$ which does not satisfy any of the congruences $a_i \pmod{m_i}$).

I can then prove that there is a $0 < U \leq 2^k$ for which $U \neq a_i \pmod{m_i}$.

I have proved Stein's conjecture and collected $10 from Erdős. The best I can prove for Erdős's conjecture is $0 < U < 2^k$.

Selfridge
Given the field $D$ of all meromorphic functions, can the subring $E$ of all entire functions be characterized by algebraic properties?
(Rubel-Shields) Consider the lune \( \Lambda = \{ z : 1 < |z|, 1 - \frac{1}{|z|} > \frac{3}{4} \} \)

If \( f \) is bounded and holomorphic (analytic) in \( \Lambda \), do there exist polynomials \( p_n \) and \( q_n \) such that

\[
\text{i)} \quad |p_n(z) + q_n(z) + \frac{1}{z}| \leq M \quad \text{where} \quad M < \infty
\]

\[
\text{ii)} \quad p_n(z) + q_n(\frac{1}{z}) \text{ converges pointwise in } \Lambda \text{ to } f.
\]
(D.S. Kahn): Trivially, \( D[X] \cong D'[X] \rightarrow D \cong D' \) if \( D \) and \( D' \) are field. How about if \( D, D' \) are integral domains or even rings?

5. Feb. 1970. (Enoch and others at U. of Kentucky have considered this problem. \( D \cong D' \) if \( D' \) is generated by its units.)

Let \( A \) be a 1-dimensional affine domain / \( k \), \( k \) arbitrary field.
If \( B \) is a ring, \( x_1, \ldots, x_n \) indeterminants over \( A \), \( y_1, \ldots, y_m \) indeterminants over \( B \) and if \( \varphi: A[x_1, \ldots, x_n] \rightarrow B[y_1, \ldots, y_m] \) as an isomorphism, then \( A \) is isomorphic to \( B \). If \( A \) is not \( k[t] \) some \( k \neq k \), then \( \varphi A = B \).

14. Dec. 73. Hochster (PAMS) has found commutative rings \( D \) and \( D' \) which are not isomorphic, but whose polynomial rings are.

[Note: The handwriting is slightly scrawled and the text is difficult to read in some parts.]
(Perronini) Is every closed convex bounded set of constant width in a real Hilbert space a sphere?

Answer: No by N. T. Hamilton
(28 August 1963)

Shown: unsolvable even for "reduced" models of ETA in which nothing equivalent to 1 appears.

\[ p_{i1}, p_{i2}, \ldots, p_{in} \text{ is not reduced even though } x_i > 0 \text{ for only finitely many } i, \]

since \( p_{ik} = 1 \) for some \( k \).

Clearly if it is unsolvable for elementary NT it is unsolvable for algebraic NT, i.e., UFD's, in 9
Proposed by Martin Gardner: In how many ways can n stamps be folded? The stamps are assumed to be in a long coil, not in a rectangular or irregular array.

2 stamps: 1 way \( \sqrt{2} \) (\( = 2^{1/2} \)) 12

3 stamps 2 ways \( \sqrt[3]{2} \) (\( = 3^{1/3} \)) 312

\( \frac{1}{\sqrt{2}} \) (\( = 3^{1/2} \)) 173

4 stamps 5 ways \( \sqrt[4]{3} \) 1231

\( \frac{1}{\sqrt[3]{2}} \) 1243

\( \sqrt[3]{2} \) 2143

\( \sqrt[3]{4} \) 4123

5 stamps around 25 ways

See Scientific American August, '63 for further details.

Sci Am Sept '63 gives:
In how many distinct ways can an $n \times m$ map be folded into a $1 \times 1$ square?

1 \times 1 
1 way 
\[ \frac{1}{n} \]

1 \times 2 
1 way 
\[ \frac{1}{4} = \frac{1}{2} \]

2 \times 2 
4 ways 

All other arrangements are either impossible to attain or reverse of the above figures. Note that the problem for the $1 \times m$ is the same as the preceding problem.
Let $\alpha^*$ be the distribution of fixed points of $\Psi^*$, i.e., $\alpha^*(n) = \text{card}\{a \in \mathbb{N}: a \leq n, \Psi^*(a) = a\}$.

(3) Prove $\alpha^*(x+y) \leq \alpha^*(x) + \alpha^*(y)$.

Relate this to $\Pi^*(x+y) \leq \Pi^*(x) + \Pi^*(y)$.
Eggen - Erdös. Let $m > \frac{c}{2}$. In the unit sphere of the unit sphere in an $m$ dimensional Hilbert space the union of fewer than $m$ sets of diameter $< 2 - \varepsilon^2$. 
Let \( a_1 < a_2 \ldots \) be an infinite sequence of positive density. Denote by \( f(x) \) the number of \( a_i/a_j \), \( a_j < x \). Is it true that \( \lim f(x)/x = \infty \) ? \( \lim f(x)/x = \infty ? \)

It can be shown rather easily that if the a sequence has positive upper logarithmic density, then \( \lim f(x)/x = \infty \). Thus, there is an affirmative answer to the second question under hypothesis much weaker than the existence of natural density.

However, a sequence may be constructed which has logarithmic density \( \frac{1}{x} \) and \( \lim f(x)/x = 0 \). Thus, the log density approach seems inadequate to answer the first question.
Let \( n_1 < n_2 < \ldots \) be an infinite sequence of integers. Is it true that
\[
\sum_{k=1}^{\infty} \frac{1}{2^{n_k-1}}
\]
is irrational?

Is the same true for
\[
\sum_{n=2}^{\infty} \frac{1}{n^{n-1}}
\]

Let \( m_k/k \to \infty \). Give \( \sum_{k=1}^{\infty} \frac{m_k}{2^m_k} \) is irrational. De Bruijn + I showed this if
\[m_k > a k \sqrt{\log k \log \log k}.\]
Denote by $m(m)$ the smallest integer $n$ so that there are $m(m)$ sets $A_i$, $1 \leq i \leq m(m)$, each $A_i$ has $m$ elements so that there is one set $G$ for which $A_i \not\subset G$, $A_i \cap G$ non-empty for all $1 \leq i \leq m(m)$.

Trivially $m(2) = 3$. Further $m(3) = 4$. $m(4)$ is unknown. W. Schmidt showed $m(m) > 2^m (1 + O(\frac{1}{m}))$ and Erdős showed $m(m) = \mathcal{O}(m^2 2^m)$.

(See Erdős, Math. Medihrift 1963)
P. Erdős and L. Moser. Let $a_1 < a_2 < \ldots < a_n$ be a sequence of integers. Let $f(n; a_1, \ldots, a_k)$ be the number of solutions of $a = \sum e_i a_i$, $e_i = 0$ or $1$. Is it true that $f(n; a_1, \ldots, a_k) < C \frac{k}{\log k}^{3/2}$? We can prove $C \frac{k}{\log k}^{3/2}$.

In connection with this, the following question can be raised. Let $0 < a_1 < a_2 < \ldots < a_k$. Does there exist a sub-sequence containing $ck$ terms where no term of the sub-sequence is the same (sum?) of any number of other terms of the sub-sequence? Perhaps $C = 1/2$. We can only show $\frac{1}{2}$ instead of $c_k$.
P. Erdős & L. Moser. If there are $n$ numbers, can we find \( \left\lceil \frac{n+1}{2} \right\rceil \) of them so that none is the same (sum?) of two others? Here, we can show \( \frac{n}{3} \) instead of \( \left\lceil \frac{n+1}{2} \right\rceil \).

We feel that this last question should be trivial, and perhaps we are overlooking something obvious.
Let $L_0$ be the lattice of open subsets of the line (ordinary inclusion, union, and intersection are the operations); similarly, let $L_c$ be the lattice of closed subsets of the line. Is there a lattice homomorphism $f: L_0 \rightarrow L_c$ that is onto?

[It is easy to see that there is no isomorphism; $L_c$ has minimal elements and $L_0$ does not.]

8/7/66: Horn has informed me that this has been settled in the negative by MacKenzie (U.C. Berkeley), for a wider class of metric spaces than just the line.
Clearly $\phi^*$ is dominated by $\phi$. What are the number theoretic properties of $\phi^*$?

After defining a submosaic in the obvious way, prove that $F(n) = \sum F(m)$, where the summation is over submosaics reflecting gen. supermult from $f$ to $F$, i.e., $F(a \cdot b) \leq F(a) \cdot F(b)$, if the mosaics of $a$ and $b$ have...
P. T. Bateman. Suppose \( g(x) \) is a polynomial with rational integral coefficients. Prove that \( g(x) - n \) is irreducible (over the field of rational numbers) for most integral values of \( n \) in some sense. For example \( x^4 - n \) is irreducible unless \( n \) is a perfect square.

A more general result was proved by Skolem in 1921 (Videnskapsselskapets Forh. I, No. 17, 57 pp.) and a still more general result by Dörge (Math. Ann. 96, 176-182) in 1926. For a simple proof of the assertion of the problem see Dörge, Math. Ann. 95 (1925) 247-256.
Anonymous, go home!

Lee Supowit

is there a function \( f \) on \( \mathbb{R} \) into \( \mathbb{R} \) such that

\[ x \in \mathbb{R} \rightarrow \exists \delta(x) > 0 \text{ such that} \]

\[ f(x) \neq f(x_0) \text{ whenever } |x - x_0| < \delta(x) \]

And \( \text{range } f \) is not countable.

Remark: I.P. Natanson says there is no such \( f \).

Verified, April 9, 1964. Natanson is right. Supowit.

James A. Marine
If $A$ is a set of positive measure, show that there is at least one subset of $A$ which is contained in an interval whose length is equal to the measure of the subset.

Parker dense set with positive measure show the assertion to be false.

An example please?

Construct on $[0,1]$ a Cantor-like nowhere dense and closed set $A$ having measure $\lambda$, $0 < \lambda < 1$.

By removing intervals of the right type. Ref: Goffman - Real Functions or Royden - Real Analysis.

If $R = \mathbb{R}^3$, consider $R - \bigcup_{n=1}^{\infty} I_n$, where $I_n = \left(\left(\frac{1}{2^n}, \frac{1}{2^{n+1}}\right)\right)$ and $\sum_{n=1}^{\infty} \frac{1}{2^n}$ on the natural.
Consider the recurrence

\[ x_{k+1} = t x_k - d x_{k-1}, \quad x_0 = a, \quad x_1 = b. \]

Assume \( |t| > 2 \). Let \( m \) be a fixed integer. Then apart from certain trivial exceptions (such as \( (a, b, m) > 1 \)) prove that one can always find \( k \) such that \( (x_k, m) = 1 \).
Let $\Gamma_t$ be the $t \times t$ unimodular group of rational integral matrices. Let $\Gamma_t^n$ be the fully invariant subgroup of $\Gamma_t$ generated by the $n^{th}$ powers of all elements of $\Gamma_t$, and $\Gamma_t(n)$ the normal subgroup of $\Gamma_t$ defined by $A \equiv I \pmod{n}$, $A \in \Gamma_t$. Is it true that for $t > 2$, $\Gamma_t^n$ is of finite index in $\Gamma_t$? Is it true that $\Gamma_t^n \subset \Gamma_t(n)$?

Seibert

Assume $\Gamma_t$ is one of a simplest forms $axb$. Let $n = 2$

then $(axb)^n = (axb)(n)$

$(axb)^2 = (axb)(2)$

$(axb)^2 = (axb)(2)$

If $\Gamma_t$ is not of the form $\Gamma_t(n)$, then the answer is no.
Compute the permanent (also determinant) of the matrix \( (\tilde{c}^i)^{ij} \), \( \tilde{c} \) a primitive \( n \)-th root of unity. Alternatively, determine the total number of permutations \( \sigma \) of the integers

\[ 1, 2, \ldots, n \] such that

\[ \sum_{R=1}^{n} R \sigma(R) \equiv 0 \pmod{n} \]

(The problems are equivalent)
Can every doubly stochastic matrix (one with row and column sums all 1) be written as the product of elementary $2 \times 2$ d.s. matrices, i.e. of matrices of the form
\[
P \left\{ \left( \begin{array}{cc} \alpha & \beta \\ \gamma & \delta \end{array} \right) + \mathbf{I}_{m \times n} \right\}^Q
\]
where $P, Q$ are permutation matrices and $\alpha + \beta = 1$?
(L.A. Rubel) Let $G$ be an open set in the complex plane, and let $P_n$ denote the class of polynomials restricted to $G$. Recursively define $P_n$ by specifying $P_{n+1}$ as the class of functions on $G$ that are pointwise limits on $G$ of a sequence of functions from $P_n$. The class $P_n$ has been characterized [1*J]. Find a characterization of the class $P_n$ for any positive integer $n$. Solve the corresponding problem for any ordinal number $n$. In particular, find a characterization of $P_2$.

By an almost disjoint family (adf) we mean a class of subsets of the natural numbers any two of which have finite intersection. Let $C_0, C_1, \ldots$ be any countable family of subsets of the natural numbers. Must there exist an uncountable adf $\mathcal{F}$ such that $f_1, f_2 \in \mathcal{F} \Rightarrow$ there is no $C_j$ for which $f_1 \subseteq C_j \subseteq f_2 - (f_1 \cap f_2)$?

Such a family always exists.
Let $S$ be an open bounded convex set in $\mathbb{R}^n$. For each $p, q \in S$ the line through $p, q$ will intersect the boundary of $S$ in $x, y$ as shown, so the order of the four points is $x, p, q, y$. Denote the Euclidean distance in $\mathbb{R}^n$ by $p$, and define

$$d(p, q) = \left| \log \frac{p(q, x)}{p(p, x)} \frac{p(p, y)}{p(q, y)} \right|.$$ 

Is $d$ a topological metric on $S$? (This is known to be true if $S$ is the interior of an ellipse, since then $S$ becomes the hyperbolic plane with metric $d$.)

R.L. Bishop, 1975. Yes. The property of $d$ in question is the triangle inequality. We use the fact that the cross-ratio is a projective invariant, in particular, it is invariant under perspectivities from various centers. Consider a triangle $p, q, r$. It is enough to do the plane case.

Let $x, y$ be the ends of $pq$.

\[ z, w ------- \rightarrow r \]
\[ u, v ------- \rightarrow r \]

In the projective plane the lines $xz$ and $yw$ meet in $x$, $uz$ and $wv$ meet in $b$.

We use the perspectivity centered at $a$ and $b$ to project $pq$ isometrically onto $rq$, with $q$ fixed and $p$ going to $p'$ on the ray $\rightarrow pq$; and to project $pr$ onto $rq$ so that $p$ goes onto $p''$ with $r$ fixed and between $p''$ and $q$. (over)
The proof is completed by showing that \( p' \) is either equal to \( p'' \) (this happens if and only if \( a = b \) and the triples \( z, x, u \) and \( z', x', u' \) are collinear) or between \( p'' \) and \( q' \). By a preliminary choice of the line at infinity, it is possible to reduce to the case pictured, where \( c \) and \( d \) are not at infinity and are on the same side of \( pq \) as \( r \) is. The argument is straightforward but messy use of plane-ordering (a line splits a plane into two halves with some obvious properties; cf. Bruck & Szmielew, Foundations of Geometry).

As a consequence of the condition for triangle equality, the triangle inequality is strict for noncollinear triangles if and only if the boundary of the convex body does not have two coplanar, noncollinear line segments.
Is there any function $F(x)$ such that
\[ \int_{0}^{x} \frac{dt}{\sqrt{1 + t^3}} = F(x) + \text{constant}. \]

Meaningless as it stands.

Of course, look in any book on elliptic integrals. See "Handbook of Integrals" for ex.

\[ 10 \text{ Nov. 78} \]

"Clyde A. Bridges"

\[ \text{Sjol dt} \]
See Moser.

Any set of (open) squares of total area $1$ can be placed in a square of area $2$ without overlap.

(i) Is the same true of any set of rectangles of total area $1$ and largest edge $1$?

(ii) What is the corresponding sharp result in higher dimensional space?

Can the rectangles of side $\frac{1}{n}$, for $n=1, 2, 3, \ldots$ all be placed without overlap in a single unit square?

Do there exist integers $a_1 < a_2 < a_3$ and $b_1 < b_2 < \ldots < b_n$ (for every $n$) such that $a_i + b_j$ is a perfect square for $i=1, 2, 3$ and $j=1, 2, \ldots, n$?

D. Kleitman (Calg. Int. Conf. on Combinatorial Structures & Their Applications, 1970) showed

\[ \frac{2^n}{n^{3/2}} \leq \text{this #} \leq \frac{2^n}{n^{5/2}} \]

Reid & Parker (J. Comb. Theo. 9, 225-238, 1970) showed (contrary to a conjecture of Lovász) $g(n) \geq \lceil \log_2 \left( \frac{16n}{7} \right) \rceil$ for $n \geq 14$.

A round-robin tournament on $n$ players involves $n(n-1)/2$ games each of which ends in a win for one of the players. Obtain asymptotic estimates for the number of distinct sets of ordered vectors $(s_1 \leq s_2 \leq \ldots \leq s_n)$ which can arise as scores in such a tournament.

Let $g(n)$ be the largest number such that every tournament on $n$ players contains a transitive tree subtournament on $g(n)$ players. It is conjectured that $g(n) \sim \log n$. 

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Is it true that for all $n$ sufficiently large there exist $n$ consecutive integers each of which has two prime factors in common with some other member of the set.

Can one place for any $n > 2$ the integers $1, 2, \ldots, (n^2)$ on the $(2n)$ intersections of $n$ lines in the projective plane in such a way that the sum of numbers on each line is the same?

Can one, in general, get by with fewer than $n^3$ multiplications in computing the product of two $n 	imes n$ matrices?

Estimate the maximal number $f(n)$ of points which can be placed in a unit cube so that the distance between any pair is at least 1. $f(1) = 2$, $f(2) = 4$, $f(3) = 8$ and $f(4) = 17$. What is $f(5)$?

If $E_i$, $i = 1, 2, \ldots, n$ is a sequence of $\pm 1$'s and $T(k) = \frac{1}{k} \sum_{i=1}^{n-k} E_i E_{i+k}$, does $\max_{E_i \in [-1,1]} \min_{0 \leq k \leq n} T(k) \to \infty$ with $n$?
Do infinitely many powers of 2 miss the digit 1 in their decimal representation?

Are there an infinity of n such that in base 3 their digits are 0's and 1's and in base 5 their digits are 0's, 1's and 2's?

\[ \frac{1}{1} + \frac{1}{2} + \cdots + \frac{1}{(p^n - 1)} \equiv 0 \pmod{p} \]

for more than 2 values of \( p \) prime? Prove or disprove.

Given a fixed constant \( c \), there exists a positive integer \( m \) such that \( \sum \max |a_{ij}| = c \) and \( n > m \), then the system of equations \( \sum_{j=1}^{m} a_{ij}x_j = 0 \), \( i = 1, 2, \ldots, m \) has a non-trivial solution with \( \|x\| < \frac{c^m}{m^{-m}} \).

Find or estimate the maximal number of \((0,1)\) \( n \times n \) matrices which commute in pairs.

Find or estimate the maximal number of \((0,1)\) \( n \times n \) matrices having exactly one 1 per row which commute in pairs.

Find or estimate the maximal number of ordinary graphs on \( n \) vertices whose incidence matrices (vertices vs. vertices) commute in pairs.
If \( 2 \cdot 3 \cdot 4 \cdots n = n! \)
and \( 1 \cdot 2 \cdot 3 \cdots n = n! \)
and \( 2 \cdot 4 \cdot 6 \cdots 2n = 2^n n! \)

What is

\[ 1 \cdot 3 \cdot 5 \cdots 2n+1 = ? \]

\[
2^n = \prod_{i=1}^{n} 2i, \quad n! = \prod_{i=1}^{n} i
\]

\[
\prod_{i=1}^{n} (2i+1) = \frac{(2n+1)!}{2^n n!}
\]
Take the sequence of ascending primes and construct a different table of the absolute values of the differences, thus:

\[
\begin{array}{cccc}
2 & 3 & 5 & 7 \\
3 & 5 & 7 & 11 \\
5 & 7 & 11 & 13 \\
7 & 11 & 13 & 17 \\
11 & 13 & 17 & 19 \\
\end{array}
\]

Question: does this diagonal contain only ones after the first column?

S. Chilton

Note: a number of us tried for all the primes \( \leq 1000 \). It works that way, at least barring any arithmetic mistakes we may have made.

The conjecture holds at least for all primes \( < 792,722 \) (see R.B. Killgrove and K.E. Ralston, "On a Conjecture Concerning the Primes," Math. Tables Aids Comput. 13(1959), 121-122)

Karl K Norton
Does there exist a group of cardinal number $|G| > \aleph_0$ which has only $|G|$ subgroups?

Rotman: If $G$ is abelian with $|G| = m > \aleph_0$, then $G$ has $2^m$ subgroups.

One needs the existence of a subset $X$ of $G$ satisfying:

1. If $[x]$ is the subgroup generated by $x$, then every cyclic subgroup of $G$ meets $[x]$.

2. If $y_1 \neq y_2 \in X$, then $[y_1] \neq [y_2]$.

When $G$ is abelian, existence is provided by maximal independent subset, where $X$ independent means $\sum_{x \in X} mx = 0$ implies $\sum_{x \in X} mx = 0$; $x, m \in \mathbb{Z}$.

One could complete the nonabelian case similarly if he could find an $X$ satisfying (1) and (2).
If \( s_1, s_2, \ldots \) is a sequence of real numbers such that
1) \( |s_{n+1} - s_n| \leq \frac{1}{n} \) and
2) for all \( k > 1 \), the subsequence \( s_k, s_{2k}, s_{3k}, \ldots \) converges.

Is it true that the sequence \( s_1, s_2, \ldots \) also converges?

Can show that for all \( k, h \),

\[
\lim_{n \to \infty} s_{kn} = \lim_{n \to \infty} s_{hn}
\]
Let $\pi$ be an equivalence relation of finite index on the set $\mathcal{T}$ of positive integers such that for all $x, y \in \mathcal{T}$

$$x \equiv y \implies (2x \equiv 2y) \land (2x+1 \equiv 2y+1)$$
$$\land (3x \equiv 3y) \land (3x+1 \equiv 3y+1)$$
$$\land (3x+2 \equiv 3y+2)$$

Conjecture:
There exist $N, k$ such that for all $x, y > N$,

$$x \equiv y \iff x = y \ (k)$$
T. McLaughlin. The following is a problem of G. E. Sacks:

Does there exist a \((\omega_1)\) countable set of \((\omega_1)\) countable linear orderings which is independent, i.e., no one of them is order-embeddable in any finite order sum of others? (The difficulty in the problem is due to the vast horde of possible embedding functions)

Liner dependant which

2 independent

one of them is order

impossible in any

finite order sum of

other (the determinant

in the function is due to

function)
Independence Questions.

1. Can it be shown in ZF (i.e., Zermelo-Fraenkel set theory without the ax. of choice) that every compact metric space is separable? (I would guess the answer is no. The natural proof seems to rely quite critically on the countable ax. of choice.)

2. Can it be shown in ZF that an uncountable subset of a separable metric space has a condensation point? (Even in the simple case of Baire nullspace $\mathbb{N}^\mathbb{N}$, it seems necessary to assume the statement (which is independent of ZF) that countable unions of countable sets are countable.)
Let $\mathbb{N}$ be the set of all natural numbers, and let $C, C_1, C_2$ refer to collections of finite subsets of $\mathbb{N}$.

Defn. $C$ is dense if for every infinite set $A \subseteq \mathbb{N}$ contains some element of $C$ as a subset.

Defn. $C$ is extendible if for every set $D \subseteq C$ we have $(D \cup \{x \times 3\}) \subseteq C$ for all but finitely many $x \in \mathbb{N}$. (The finite number of exceptional $x$ may depend on $D$.)

Conjecture If $C_1, C_2$ are each dense and extendible, then there is a set $D$ which belongs to each of $C_1, C_2$.

(Note: If the conjecture is weakened by adding the following hypothesis, it becomes equivalent to Ramsey's theorem:

There is a (fixed) number $n$ such that for every infinite set $A \subseteq \mathbb{N}$, $A$ contains as set of cardinality at most $n$ which belongs to $C_1 \cup C_2$.

Note: (May 14, 1968) This conjecture has now been proved by Galvin and Prikry.
For \( f(x,y) \) smooth, define

\[
\tilde{f}(x,y) = \text{P.V.} \int_{-\infty}^{\infty} \frac{f(x-t, y-t^3)}{t} \, dt.
\]

Is \( f \to \tilde{f} \) continuous on \( L^p(\mathbb{R}^2) \) for \( p \neq 2 \)? (This means: there is a \( c_p \) such that \( \|\tilde{f}\|_p \leq c_p \|f\|_p \)).

What is the side of the equilateral triangle of least area that can accommodate each arc of length 1?

If this is too easy — what is the perimeter of the triangle with given angles and least perimeter that can accommodate each arc of length 1?

Of all such triangles, is the equilateral the smallest?
For the set $x = \pm 8, \; y = \pm 3$ the only solution in integers for the
(Diophantine) equation

\[ 5(y^2-1) = x^2 + 256 \]

Edward Lucas in his Théorie des Nombres p. 150 (1891 edition) writes,

Verify the formula

\[ \frac{a-b}{c} + \frac{b-c}{a} + \frac{c-a}{b} \]

then

\[ \frac{c}{a-b} + \frac{a}{b-c} + \frac{b}{c-a} = 9 \]

On supposing \[ a+b+c = 0 \]

What is the "trick" that will give a quick solution?

The trick is to multiply out and set

\[ \left( \frac{a}{b-c} \right) \left( \frac{b}{c-a} \right) = x, \; \left( \frac{b}{c-a} \right) \left( \frac{c}{a-b} \right) = y, \; \frac{c}{a-b} \frac{a}{b-c} = z \]

so that \[ xy = 2 \]

Then \[ x + (y + 1)(z + 1) + 1 = 9 \]

So the end

\[ x \, y \, z = 1 \]

Knowing \[ x, y, z = 2 = 1 \]

\[ z(x) = 2 \]

\[ y = 1 \]

\[ z = 1 \]

\[ x = 1 \]

\[ (1 + 1 + 1)(1 + 1 + 1) \]

\[ (3)(3) = 9 \]
17 Oct. 1973

Recently, Prof. Bateman established (Problem E2051, Amer. Math. Monthly 76 (1969), 170-172) that the arithmetic function \( f_8(n) = (2^n - 1) \) is multiplicative iff \( s \in \{1, 2, 4, 8\} \), where \( r_s(n) \) is the number of representations of \( n \) as a sum of \( s \) integral squares. Prove the stronger result that \( f_8(\cdot) \) is generalized multiplicative (for a definition see Amer. Math. Monthly 72 (1965), 1140-1141) if and only if \( s \in \{1, 2, 4, 8\} \). Apply analogous methods to generalize the Bateman-Lagrange theorem, i.e., \( f_8(\cdot) \) is generalized multiplicative iff \( s \in \{1, 2, 3, \ldots, 8\} \). See Bateman et al., Notices (Jan. 1973).

Consider the following synthesis of the Browder-Schäuder fixed-point Th. & Minkowski Th. on the existence of lattice points: Determine necessary sufficient conditions on a continuous transformation \( T \) of a compact set of \( \mathbb{R}^n \) into itself in order that \( T \) has a fixed lattice point distinct from the Origin.
17 Oct. 73 Prove that there is a polynomial (over \( \mathbb{Z} \)) of degree at most 21 in 2 variables recursive whose range is precisely the set of all prime numbers.

2 Oct. 73 Prof. Moser, among many others, showed that every real-valued monotone multiplicative arithmetic function \( f(1), f(n) \neq 0 \), satisfies \( f(n) = n^\alpha \) for some constant \( \alpha \). Prove the stronger result: every real-valued monotone generalized multiplicative arithmetic function \( f(1), f(n) \neq 0 \), satisfies \( f(n) = n^\alpha \) for some constant \( \alpha \). (See: Canadian Math. Bull. 9 (1966), p. 115, Prob. 112.)
A. S. Fraenkel
(~ 1974)

\[ d_i, \ldots, d_m \text{ rational, } r_i, \ldots, r_m \text{ rational. Suppose that } \{ d_i + r_i i \} \text{ face complementing once in exactly one of the sequences } \{ r_i + r'_i i \}. \]

Prove or disprove: \( d_i / d_j = k \) for some \( i \neq j \) given \( r_i \) is not \( r_j \).

\[ \text{In this connection, consider the complementing system } \left[ \frac{2^n - 1}{2^{m-k}} \right] + 1 - 2^{-k} \]

For \( k = 1, \ldots, m, \quad n = 1, \ldots \). Erdős asked whether this is perhaps the only system for which all the moduli \( d_i \) are different (\( m \geq 3 \)).


1. Special proofs of special cases of the conjectures (note that (i) \( \rightarrow \) (ii)) have appeared in Fraenkel J Comb Theory A 14 (1971) 8-20.

2. The validity of (i) \( \Rightarrow \) (ii) has been shown by Graham J Comb Theory A (15) (1973) 354-358. (Two moduli must be equal, as in the integer case.)

3. Special cases of both conjectures (rational moduli) were proved in Berger, Felzenbaum, Fraenkel, J Comb Theory A 42 (1986) 150-153.

4. Morikawa, Bull. Liberal Arts, Nagasaki Univ. 26 (1985) 1-13, motivated by the conjectures, has given a proof of the "Japanese Remainder Theorem" giving a necessary and sufficient condition for the disjointness of two rational density sequences. (The special case when both denominators are 1 is the Chinese Remainder Theorem.) Simpson, Tech Report 3/90, Curtin Univ., Perth WA (1990) has given a more transparent proof. He and Vera Sós are considering to write up a proof based on the 3-gap Theorem.

5. Simpson, Nitsch Math (3 1971) has proved Conj. (i) for the special case when the smallest rational modulus is \( \leq 2 \). \( * \) Terminology due to Simpson.

6. The general case of both conjectures is still open for the case of rational moduli, though it is settled for both the integer and irrational cases.
G S. Schreier  
July 75

1. Is every countable metabelian group embeddable into a free group?

2. If $G$ is a countable group all of whose finitely generated subgroups are 3-transitive groups, is $G$ embeddable into a finitely presented group?

In the case where all the di are irrational.

Preparation

[signature]
Prove

\[
\lim_{n \to \infty} e^{-ny} \left(1 + \frac{ny}{1!} + \frac{(ny)^2}{2!} + \ldots + \frac{(ny)^{n-1}}{(n-1)!}\right) = \begin{cases} 
1 & 0 \leq y < 1 \\
\frac{1}{2} & y = 1 \\
0 & 1 < y
\end{cases}
\]

2. Gerrard 1977

Conjecture: If

\[ S^2 \xrightarrow{f} S^2 \xrightarrow{g} \mathbb{E}^2 \]

with \( f \) and \( g \) continuous, \( \exists x \) such that \( g(x) = g(f(x)) \)
General case. (I need this solved)

$b_i(t)$, $d_i(t)$, $\mu_i(t)$, $\mu_2(t)$ nonnegative analytic functions of $t$ for $t \geq 0$, $Q = Q(s_1, s_2, s_3, t)$.

Solve the following partial differential equation.

$$\frac{\partial Q}{\partial t} = \left\{ s_1^2 b_1(t) + s_2 (s_2-1) \mu_1(t) + s_1 (s_3-1) \mu_2(t) \\
- s_1 \left[ b_1(t) + d_1(t) \right] + d_1(t) \right\} \frac{\partial Q}{\partial s_1}
$$

$$+ \left\{ s_2^2 b_2(t) - s_2 \left[ b_2(t) + d_2(t) \right] + d_2(t) \right\} \frac{\partial Q}{\partial s_2}
$$

$$+ \left\{ s_3^2 b_3(t) - s_3 \left[ b_3(t) + d_3(t) \right] + d_3(t) \right\} \frac{\partial Q}{\partial s_3}$$

$Q(s_1, s_2, s_3, 0) = s_1^{m_{10}} s_2^{m_{20}}$, $m_{10}$ and $m_{20}$ are fixed constants.

(2) Special case.

If the above PDE is not solvable, try to solve the special case:

$b_i(t) = b_i$, $d_i(t) = d_i$, $\mu_i(t) = \mu_i$ and $\mu_2(t) = \mu_2$ all independent of $t$.
Let $n_1, n_2, \ldots, n_k$ be relatively prime square-free integers. Let $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_k$ be any sequence of $\pm 1$'s. Is it true that there exists a prime $p$ such that
\[
\left( \frac{n_i}{p} \right) = \varepsilon_i \quad i = 1, 2, \ldots, k.
\]
Mary Student Johnson  
1984  
October 8  

March 2, 1984  

As empty space or no time constant with a null set i.e., nonexistent or nullification of previous time or time which might have been as time rearranges the above diagram occurs:

- Time splits apart leaving a hole which represents empty space for no time.
- The fabric on each side appears to break into xT b's which recur in

Diagram 1

Theory of the Fabric of Time

(empty space or no time)
the 2 of the curvature of time is consistent with the pt of segment from the parabolic hole.

in realignment the fabric intertwines as slips under each other to form the new fabric of time

similar to liquid crystals

Nobel Prize Nominee, 1983 Medicine 
Nobel Prize Nominee, 1984 Medicine
Nobel Prize Nominee, 1985, 86 Medicine, Chemistry
Nobel Prize Winner 1986 awarded Chemistry

Pulitzer Prize Winner 1987
awarded Jan in Feb 1987

March 1989

Parallel existence or time paths is a reality

Time through the parabolic hole can alter parallel existences into the same one

or

Two parallel time paths may merge
Starting with the affirmation of man I work my way backwards using cynicism the time monitor the space measure live sweat dream eight years the tide the rise and the fall
11-6-89

1. Bromley Hall Applied
N. Boston
August, 1990

Proved by
twad mosul
See J. Alg. 1991

1) Let $G$ be a group and $A = \text{Aut}(G)$. Given subgroup $H$ of $G$ let $H^* = \{x \in A: h^x = h \forall h \in H\}$.

Given subgroup $B$ of $A$ let $B^* = \{g \in G: g^f = g \forall f \in B\}$.

Call $G$ Galois-theoretic (GT, for short) if both $^*$ maps are bijective.

Claim: Are the only GT groups $\mathbb{Z}_3$, $C_3$, $S_3$, $\mathbb{Z}_2$. Note: Marty Isaacs has shown that a finite GT group must be solvable.

2) Let $d(G)$ denote the number of generators of a finite group $G$. A q.f important in profinite group theory (due to Ribes et al.) is whether, given 2 finite groups $G, H$, there exists a finite group $K$ containing copies of $G$ and $H$ and generated by these copies, such that $d(K) = d(G) + d(H)$.

3) Baker et al. have asked whether there is a 5th term in the sequence 1, 3, 8, 120, i.e. an integer which when multiplied by 1, 3, 8, or 120 gives a square minus one. (This problem has a lot of algebraic structure to it, maybe surprisingly.)

Sorry, this is known. See e.g. Fibonacci Quart 17.
Use any sequence $c_1, c_2, c_3, \ldots$ of 0's and 1's to define a repetition-resistant sequence $s = (s_1, s_2, s_3, \ldots)$ inductively as follows:

(i) $s_1 = c_1$, $s_2 = 1 - s_1$

(ii) for $n > 2$, let

$$L = \max \{ i \geq 1 : (s_{n-1+i}, \ldots, s_m, 0) = (s_{n-1+i}, \ldots, s_m, 0) \text{ for some } m < n \}$$

$$L' = \max \{ i \geq 1 : (s_{n-1+i}, \ldots, s_m, 1) = (s_{n-1+i}, \ldots, s_m, 1) \text{ for some } m < n \}$$

(set that $L$ is the maximal length of tail-sequence of $(s_1, s_2, \ldots, s_n, 0)$ that already occurs in $(s_1, s_2, \ldots, s_n)$, and similarly for $L'$), and

$$s_{n+1} = \begin{cases} 0 & \text{if } L \leq L' \\ 1 & \text{if } L > L' \\ c_n & \text{if } L = L' \end{cases}$$

Prove or disprove: $s$ contains every binary word.

Example: If $c_i = 0$ for all $i$, then

$$s = (0, 1, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 1, 0, 0, 1, 1, 1, 0, 1, 0, 0, \ldots)$$